647510 **BSC2MatC201x** **Seat No :________**

B.Sc. Semester - 2 (*CBCS*) Examination

March/April – 2024 (As Per Syllabus Year-2018)

MATHEMATICS (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any two out of three. (10)

(1) Find centre and radius of the circle $x^2+y^2+z^2-x+z-2=0$; $x+2y-z=4$.

(2) Give the relationship among spherical, cartesian and cylindrical co-ordinate systems in $\mathbb{R}^3$. Also convert the point (1,2,3) in to spherical and cylindrical systems.

(3) Show that the plane $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ and find the point of contact.

Que.1(B) Answer any one out of two. (04)

(1) Find the equation of the sphere passing through the points (0,0,0), (-a,b,c), (a,-b,c), (a,b,-c).

(2) Find equation of cylinder whose generator is parallel to $\frac{x}{2}=\frac{y}{3}=\frac{z}{4}$ and passing through $x^2+xy+y^2=1$, $z=0$.

Que.2(A) Answer any two out of three. (10)

(1) If $z(x+y)=(x^2+y^2)$, show that $\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)^2 = 4\left(1-\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)$.

(2) If u= f(x,y) and $x=r\cos\theta$, $y=r\sin\theta$ then prove that

$$\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2=\left(\frac{\partial u}{\partial r}\right)^2+\frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$$

(3) If f(x,y)=0 then obtain $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$.

Que.2(B) Answer any one out of two. (04)

(1) If $w=\frac{e^{x+y+z}}{e^x+e^y+e^z}$ then find the value of $\frac{\partial w}{\partial x}+\frac{\partial w}{\partial y}+\frac{\partial w}{\partial z}$

(2) Find $\frac{dy}{dx}$ for the implicit function $\sin(xy)= e^{xy}+ x^2y$.

Que.3(A) Answer any two out of three. (10)

(1) If $u=\frac{x+y}{1-xy}$, $v=\tan^{-1}x+\tan^{-1}y$, then find $\frac{\partial(u,v)}{\partial(x,y)}$

(2) Expand log xy in power of x-1 and y-1.

(3) If $u=x+y+z$, $uv=y+z$, $uvw=z$ then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

Que.3(B) Answer any one out of two. (04)

(1) If $u=x^2-2y$, $v=x+y+z$, $w=x-2y+3z$ then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

(2) If $f(x,y) = x^3+xy+y^3$ then find the approximate value of f(1.01,2.98).

Que.4(A) Answer any two out of three. (10)

(1) Prove that every square matrix can be expressed uniquely as the sum of a symmetric and skew symmetric matrix.

(2) In usual notation, prove that $(A+B)^T=A^T+B^T$ and $(AB)^T= B^T A^T$.

(3) Find rank of matrix $\begin{bmatrix} 3 & 1 & 4 & 0 \\ 0 & 2 & 2 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$ using echelon form.

Que.4(B) Answer any one out of two. (04)

(1) If A= $\begin{bmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{bmatrix}$ is orthogonal then find value of l,m,n.

(2) Find rank of matrix $\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$

Que.5(A) Answer any two out of three. (10)

(1) State and prove Cayley- Hamilton theorem.

(2) Verify Cayley- Hamilton theorem for $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

(3) Using Cayley- Hamilton theorem find inverse of $\begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$.

Que.5(B) Answer any one out of two. (04)

(1) Find eigen values and eigen vectors of matrix A= $\begin{bmatrix} 5 & 3 \\ -1 & 1 \end{bmatrix}$.

(2) Find eigen values and eigen vectors of matrix A= $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$.

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